Nonlinear Effects of Loan-to-Value Constraints*

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Abstract

This paper investigates the impact of loan-to-value (LtV) borrowing limits in equilibrium asset pricing models with occasionally binding credit constraints. These constraints give rise to a Fisherian debt-deflation mechanism, where shocks can trigger cascading effects resulting in significant declines in consumption, asset prices, and borrowing reversals—characteristic of financial crises. We identify critical thresholds that govern asset price, consumption, and current account dynamics. Our analysis uncovers a nonlinear relationship between the LtV limit and adverse effects on macroeconomic outcomes, which aligns with cross-country evidence regarding the relationship between the financial development level and the severity of consumption declines during crises.

J.E.L. classification codes: E31, E37, E52, F41, G01.

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1 Introduction

In the wake of the 1990s' sudden stops experienced by numerous emerging economies and the economic turmoil of the 2007-2008 Global Financial Crisis, the quest for models capable of elucidating the dynamics of these crises has intensified. Pioneered by the work of Mendoza (2010), one prominent avenue has been the exploration of models featuring occasionally binding credit constraints, both for normative and positive analysis. Central to these models is the incorporation of loan-to-value (LtV) type borrowing constraints, which cap borrowing at a fraction of the market value of asset holdings.

These models, enriched by the presence of occasionally binding constraints, unveil a Fisherian debt-deflation mechanism. As shown by Schmitt-Grohé and Uribe (2017), in vulnerable economic states, exogenous shocks can trigger the tightening of these constraints, precipitating a cascade of effects. As asset prices plummet in response, the constraint tightens further, exacerbating deflationary pressures. This feedback loop often manifests in sharp declines in consumption, asset prices, and borrowing reversals—hallmarks of financial crises.

However, recent research, epitomized by the work of Schmitt-Grohé and Uribe (2021), challenges conventional wisdom regarding the implications of collateral constraints. Contrary to prior intuition, these authors argue that the effects of collateral constraints may not always amplify financial disturbances. Rather, they posit that such constraints could engender dynamics seemingly at odds with earlier literature, including the emergence of multiple equilibria.

In particular, Schmitt-Grohé and Uribe (2021) highlight the nonlinearity of adjustment in economies subject to collateral constraints. These dynamics give rise to two distinct equilibria: one characterized by self-fulfilling financial crises driven by pessimistic views on collateral values, and another marked by underborrowing and excessive precautionary savings as a way to self-insure.

This paper contributes to this evolving literature by rigorously characterizing the analytical solution of equilibrium asset prices in small open economies featuring loan-to-value collateral constraints. Specifically, we investigate the ramifications of varying collateral fractions on key macroeconomic and financial variables. Our analysis uncovers that when the LtV limit is close to zero, tighter constraints induce smaller drops in consumption during crises. Conversely, when the LtV limit is close to one, we observe that tighter constraints induce larger drops in consumption during crises.

The nonlinear impact of LtV limits is influenced by a general equilibrium effect caused by asset prices. When the LtV limit rises, two conflicting outcomes occur. Firstly, increased indebtedness makes the economy more fragile, leading to a greater decline in consumption. Secondly, a more relaxed collateral constraint lessens the impact on asset prices. In economies with low LtV, the first effect is more significant, whereas in economies with high LtV, the second effect prevails.

To assess the empirical validity of our theoretical findings, we examine the cross-country evidence on how the level of financial development, which proxies the model's LtV limits, is correlated with the severity of consumption declines during a financial crisis. To do this, we construct a panel database of Sudden Stop episodes from Bianchi and Mendoza (2020), a financial development index developed by Svirydzenka (2016) and macroeconomic aggregates from The World Bank (2024). Predictive quadratic regressions validate our theoretical findings. In particular, we find a U-shaped relationship between consumption growth and financial development during crises.

Our paper is related to several strands of the literature. The investigation of financial crises, particularly within the context of emerging economies, has been significantly advanced by models incorporating occasionally binding credit constraints. These models, pioneered by Mendoza (2010), have provided essential insights into the mechanisms driving sudden stops and economic downturns. Central to this approach is the loan-to-value (LtV) constraint, which limits borrowing based on the market value of assets. Schmitt-Grohé and Uribe (2017) further developed these models by examining stock collateral constraints, revealing how exogenous shocks can tighten these constraints, leading to a Fisherian debt-deflation cycle that exacerbates financial instability.

The literature has also explored the possibility of multiple equilibria arising from these constraints. Schmitt-Grohé and Uribe (2021) analyzed flow collateral constraints, demonstrating that these models can result in nonlinear dynamics and the potential for multiple equilibria. Similarly, Jeanne and Korinek (2019) discussed the heuristic implications of stock collateral constraints, suggesting that under certain conditions, these constraints could lead to self-fulfilling financial crises or excessive precautionary savings behavior.

In addition to the theoretical contributions, various studies have employed these models to explore the broader implications of collateral constraints. For instance, Bianchi and Mendoza (2018) and Jeanne and Korinek (2019) applied similar models to investigate the role of pecuniary externalities in financial crises, while other scholars such as Lorenzoni (2008), Mendoza (2010), Bianchi (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013, 2016), and Ottonello, Perez, and Varraso (2022), examined how these constraints influence overborrowing and the design of optimal macroprudential policies. Jensen, Ravn, and Santoro (2018) studied the business-cycle implications of changing LtV limits. Devereux, Young, and Yu (2019) also contributed to this discussion by exploring the capital flow dynamics under different regulatory frameworks.

Moreover, the relationship between financial development and economic outcomes has been extensively studied. Levine (1997), Loayza and Ranciere (2006), and Bordo and Meissner (2015) explored how financial development affects economic growth, business cycles, and the severity of financial crises. These studies provide empirical support for the theoretical models, suggesting that the level of financial development can significantly influence the impact of collateral constraints on macroeconomic stability.

The remainder of the paper is organized as follows. Section 2 describes the sudden stop model with perfect foresight and characterizes its analytical solution. In Section 3 we extend the model to an environment with uncertainty. We solve the model numerically and show that our results are driven by a general equilibrium effect through the asset's price. Section 4 describes the model validation exercise and shows that our theory predictions are in line with the data. Finally, Section 5 concludes.

2 Model with Perfect Foresight

The model is similar to Bianchi and Mendoza (2018) and Jeanne and Korinek (2019). The small open economy is populated by a continuum of homogeneous households. The representative household maximizes a CRRA utility function, $u(c)$ and only derives utility from consumption, c. The household buys next period international bond holdings, b', that pay an exogenous interest rate R and also buys next period domestic asset holdings, a', with an endogenous price q. The domestic asset net supply is fixed and normalized to 1. Total endowment output in the economy is given by y, whose share of labor income is given by $(1-\alpha)$ and share to capital (dividends) income by α. Finally, the household's credit is constrained by a standard loan-to-value collateral constraint in which debt cannot exceed a constant fraction κ of the market value of next period asset holdings.[1] The problem of the household in recursive form is given by:

$$\begin{aligned} V(a,b) &= \max_{\{c,b',a'\}} u(c) + \beta V(a',b') \quad s.t. \\ c + R^{-1}b' + qa' &= (1-\alpha)y + a(\alpha y + q) + b \\ R^{-1}b' &\geq -\kappa q a'. \end{aligned} \tag{1}$$

Let $\lambda > 0$ and $\mu \geq 0$ be the multipliers on the budget constraint and collateral constraint, respectively, and define $\hat{\mu} = \mu/\lambda \geq 0$. Then, assuming the market

[1]The timing of assets used as collateral on the right-hand side of the constraint relies on assumptions about the characteristics of credit contracts and their enforcement. Both timing approaches have been widely adopted. For more details see Bianchi and Mendoza (2020) and Ottonello, Perez, and Varraso (2022).

clearing condition, $a = a' = 1$, the equilibrium conditions are:

$$\begin{aligned} u_c(c) &= R\beta u_c(c') + \mu && (2) \\ q &= \beta \frac{u_c(c')(\alpha y' + q')}{u_c(c)(1-\kappa\hat{\mu})} && (3) \\ 0 &= \mu(R^{-1}b' + \kappa q) && (4) \\ c &= y + b - R^{-1}b'. && (5) \end{aligned}$$

Where $u_c(c)$ corresponds to the first derivative of the utility function with respect to consumption. Additionally, from Eq. 2-3 we obtain the following expression for the equity premium where the return on the asset is $R^q = \frac{\alpha y' + q'}{q}$:

$$R^q - R = \underbrace{\frac{R\hat{\mu}}{1-\hat{\mu}}}_{\text{Indirect}} \underbrace{(1-\kappa)}_{\text{Direct}}. \tag{6}$$

In Eq. 6 we can see a direct negative effect of the LtV limit on the equity premium and an indirect effect that will depend on how strong the collateral constraint binds.

Now assume $b > -R\kappa q$ such that the economy is not constrained and $R\beta = 1$, to analyze a stationary equilibrium in which consumption is constant: $c = c'$. From Eq. 2-5 we obtain:

$$\begin{aligned} c = c' &= y + (1-\beta)b && (7) \\ \mu = \hat{\mu} &= 0 && (8) \\ q &= \frac{\beta\alpha y}{1-\beta}. && (9) \end{aligned}$$

The equilibrium characterized by Eq. 7-9 is a well known result. The household will keep a constant consumption equal to the total endowment, y, plus or minus the interest paid or earned depending on the sign of the initial bond position, b. The current account is constant and zero, $CA_t = b' - b = 0$, the collateral constraint is not binding by assumption and the equilibrium exhibits the standard forward looking asset price.

Now assume that the household is marginally constraint, i.e., the debt is at its highest level and the collateral constraint multiplier is zero: $b = -R\kappa q$ and $\mu = 0$. Then the equilibrium allocations become:

$$b = b' = -\frac{\kappa \alpha y}{1-\beta} \tag{10}$$

$$c = y(1-\kappa\alpha). \tag{11}$$

2.1 Analytical Results

The analytical results in this section provide a clear and concise derivation of the equilibrium allocations and asset prices in a simplified model setup with a logarithmic utility function (the coefficient of risk aversion is equal to 1). By assuming an unexpected wealth-neutral shock to the economy and a sequential framework (Eq. 12), we can explore how the LtV limit influences the outcomes of the model under different scenarios. The closed-form solutions derived here help to uncover key insights into the dynamics of asset prices, consumption, and debt, particularly when the economy faces a binding collateral constraint.

$$y_t = \begin{cases} y & \text{for } t \leq -1 \\ \gamma y & \text{for } t = 0 \text{ with } \gamma < 1 \\ \tilde{y} = y(1 + \left(\frac{1-\beta}{\beta}\right)(1-\gamma)) & \text{for } t \geq 1. \end{cases} \tag{12}$$

The solution of the model starts by considering two scenarios based on the relationship between the equilibrium asset price in period 0 and period 1. In period $t = 0$, the endowment level in the economy is lower than in the previous period and from $t = 1$ onward, the endowment is larger such that the net present value stays constant.[2] It is straightforward to see that, due to the shock, the household would like to smooth her consumption by increasing her debt in period 0. However, note that in the previous period, the household was holding the maximum possible debt. Therefore, there are two possible scenarios.

Scenario 1: If the equilibrium price in period 0 is less than or equal to the unconstrained equilibrium price in period 1 ($q_0 \leq q_1 = \frac{\beta\alpha\tilde{y}}{1-\beta}$), then, the maximum debt in period 0 is supported in an unconstrained period 1 and, hence, the household can maintain a constant consumption from period 1 onward. The equilibrium allocations are such that the household consumes as much as possible in period 0 since she is at the debt limit and, from period 1 onward, the household consumes a constant level. Note that $q_{-1} = \frac{\beta\alpha y}{1-\beta} < \frac{\beta\alpha\tilde{y}}{1-\beta}$, hence this case allows for some asset price inflation in period 0. Whenever there is asset price inflation, the current account

[2]Note that in the absence of the collateral constraint, such wealth neutral shock would not have any effect on consumption. The economy would be able to perfectly smooth consumption by adjusting the current account. During period 0, the economy would borrow from abroad and from period 1 onward, the economy would pay the interests of such borrowing with the additional endowment.

will be pro-cyclical.

Scenario 2: If the equilibrium price in period 0 is greater than the unconstrained equilibrium price in period 1 ($q_0 > \frac{\beta\alpha\tilde{y}}{1-\beta}$), then the maximum debt in period 0 is not supported in an unconstrained period 1 and hence the household cannot maintain a constant consumption from period 1 onward. The equilibrium allocations are such that the household gets as much debt as possible in period 0 as long as $c_{-1} \geq c_0$ and, from period 1 onward, the household starts a deleveraging process. In this process, the household holds the maximum amount of debt possible in each period and slowly increases her consumption until she reaches the steady state in which the constraint is marginally binding and holds a constant consumption level. Note that in this case, the effects of the shock are persistent, the current account is pro-cyclical, the asset price increases in period 0 and slowly decreases until it converges to $\frac{\beta\alpha\tilde{y}}{1-\beta}$, which is a counter-intuitive dynamic.

Now we turn to characterizing for which LtV limits we are in the first scenario. Let $z(\kappa) = \frac{c_1^{-1}}{c_0^{-1}} \in (0, 1]$. Note that $z(\cdot)$ is an implicit function of κ since the equilibrium consumption choices depend on it and, $z(\cdot) \leq 1$ since $c_0 \leq c_1$, because if not then the household could save in period 0 to smooth her consumption. From

Eq. 3 we get the following relation:

$$q_0 \leq q_1$$
$$\Leftrightarrow$$
$$\frac{\beta z(\kappa)(\alpha\tilde{y} + q_1)}{1 - \kappa(1 - z(\kappa))} \leq \frac{\beta\alpha\tilde{y}}{1 - \beta}$$
$$\Leftrightarrow$$
$$z(\kappa)(1 - \kappa) \leq 1 - \kappa.$$

The last inequality is satisfied only for all $0 \leq \kappa \leq 1$. Hence, for $0 \leq \kappa \leq 1$ we are in scenario 1 and for $\kappa > 1$ we are in scenario 2.

Assumption 1. *Let the LtV limit be such that* $0 \leq \kappa \leq 1$.

Under Assumption 1, the following equations summarize the equilibrium allocations and asset prices:

$$q_{-1} = \frac{\beta\alpha y}{1 - \beta} \tag{13}$$

$$c_{-1} = y(1 - \kappa\alpha) \tag{14}$$

$$b_0 = -\frac{\kappa\alpha y}{1 - \beta} \tag{15}$$

$$c_0 = \gamma y + b_0 - R^{-1}b_1 = y\left(\frac{\gamma(1 - \beta) - \kappa\alpha}{1 - \beta}\right) + \kappa q_0 \tag{16}$$

$$b_1 = -R\kappa q_0 = b_\tau \text{ for } \tau \geq 2 \tag{17}$$

$$c_1 = \tilde{y} + b_1 - R^{-1}b_2 = \tilde{y} - \frac{1 - \beta}{\beta}\kappa q_0 = c_\tau \text{ for } \tau \geq 2 \tag{18}$$

$$q_1 = \frac{\beta\alpha\tilde{y}}{1 - \beta} = q_\tau \text{ for } \tau \geq 2. \tag{19}$$

Note that the only unknown is the current price q_0. Combining Eq. 2-3 and subtituing Eq. 16-19, we obtain a quadratic polynomial:

$$
\begin{aligned}
& q_0 = \beta \frac{c_1^{-1}(\alpha\tilde{y} + q_1)}{c_0^{-1}(1-\kappa\hat{\mu}_0)} \\
& \Leftrightarrow \\
& q_0 = \beta \frac{c_0(\alpha\tilde{y} + q_1)}{\kappa c_0 + (1-\kappa)c_1} \\
& \Leftrightarrow \\
& q_0 = \left(\frac{\beta\alpha\tilde{y}}{1-\beta}\right) \frac{c_0}{\kappa c_0 + (1-\kappa)c_1} \\
& \Leftrightarrow \\
& q_0 = \left(\frac{\beta\alpha\tilde{y}}{1-\beta}\right) \frac{y\left(\frac{\gamma(1-\beta)-\kappa\alpha}{1-\beta}\right) + \kappa q_0}{\kappa\left(y\left(\frac{\gamma(1-\beta)-\kappa\alpha}{1-\beta}\right) + \kappa q_0\right) + (1-\kappa)\left(\tilde{y} - \frac{1-\beta}{\beta}\kappa q_0\right)} \\
& \Leftrightarrow \\
& 0 = Aq_0^2 + Bq_0 + C.
\end{aligned}
$$

Where:

$$
\begin{aligned}
A &= \frac{\kappa}{\beta}(\kappa + \beta - 1) \\
B &= \kappa y\gamma\left(1 - \frac{\kappa\alpha}{\gamma(1-\beta)}\right) + \tilde{y}\left(1 - \kappa - \frac{\beta\alpha\kappa}{1-\beta}\right) \\
C &= \frac{\beta\alpha\tilde{y}y\gamma}{1-\beta}\left(\frac{\kappa\alpha}{\gamma(1-\beta)} - 1\right).
\end{aligned}
$$

Which can be solved using the general quadratic formula solution:

$$q_0^H = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad \text{and} \quad q_0^L = \frac{-B - \sqrt{B^2 - 4AC}}{2A}. \tag{20}$$

Equation 20 is the analytical solution for the asset price on the period when the unexpected wealth neutral shock hits the economy. In the next subsection we use this analytical solution to characterize the parameter regions where the equilibrium is unique, multiple or non-existent.

2.1.1 Characterization of the Equilibrium Asset Price and Cases

In this subsection, we obtain the general cases for uniqueness, multiplicity or non-existence of the equilibrium.

Assumption 2. *The drop in the endowment is not too large:* $\gamma > \frac{\alpha}{1-\beta(1-\alpha)}$.

Assumption 2 is plausible since γ corresponds to 1 minus the size of the shock so it is expected to be close to 1 and α corresponds to the capital income share which is expected to be less than $\frac{1}{3}$. Now we obtain the following cases:

Cases:

$$
\begin{array}{rl}
\text{Unique} & \begin{cases}
A = 0 \Leftrightarrow (\kappa = 1-\beta \text{ or } \kappa = 0) \text{ then } q_0 = \frac{-C}{B} > 0 \text{ since } B > 0, C < 0. \\[2ex]
A > 0 \text{ and } C < 0 \Leftrightarrow \kappa \in [1-\beta, \min\{1,\ (1-\beta)\frac{\gamma}{\alpha}\}) \text{ then } q_0^H > 0. \\[2ex]
A < 0 \Leftrightarrow \kappa < 1-\beta \text{ then} \\[2ex]
\quad q_0^H > 0,\ q_0^L > 0 \text{ but only } q_0^L > 0 \text{ is consistent with } c_0 \leq c_{-1}. \\[2ex]
\kappa = 1 \text{ then } q_0 = \frac{\beta\alpha\tilde{y}}{1-\beta}. \\[2ex]
\kappa = (1-\beta)\frac{\gamma}{\alpha} \text{ then } q_0 = \frac{-B}{A} > 0 \text{ since } B < 0, A > 0.
\end{cases} \\
\text{Multiple (pair)} & \begin{cases} A > 0 \text{ , } C > 0 \text{ and } B^2 - 4AC > 0 \text{ then } q_0^H > 0,\ q_0^L > 0. \end{cases} \\
\text{Non-existence} & \begin{cases} B^2 - 4AC < 0. \end{cases}
\end{array}
$$

In summary, $0 \leq \kappa \leq \min\{1,\ (1-\beta)\frac{\gamma}{\alpha}\}$ guarantees a unique equilibrium. Intuitively, more volatile economies (lower γ) and economies with higher capital returns (higher α) require a lower LtV limit to guarantee a unique equilibrium. This result is important since it says that a unique equilibrium is guaranteed to exist when the LtV limit is close to zero.

Having a closed form solution for the equilibrium asset price allows us to get the following results.

Proposition 1. *With $\kappa \in [0, 1]$, then the economy has a unique equilibrium if and only if the debt-to-endowment ratio is less than or equal the shock size:* $\frac{-b}{y} \leq \gamma$.

To obtain this result, we use the bond expression in Eq. 10 divided by the endowment and note that under a unique equilibrium, then $\alpha \leq (1-\beta)\frac{\gamma}{\kappa}$. Combining the two expressions delivers the *if* part of Proposition 1. To obtain the *only if* part we start from the debt-to-endowment ratio $\left(\frac{-b}{y}\right)$ and replace the bond expression in Eq. 10. After some manipulation we obtain the uniqueness condition. QED.

Proposition 2. *When debt is not possible ($\kappa = 0$) then:*

- *One-to-one mapping from shock to drop in price and consumption, $\frac{q_0}{q_{-1}} = \frac{c_0}{c_{-1}} = \gamma$.*
- *Crisis amplification or dampening from increases in the LtV from zero depend on the capital share for the asset price and unambiguous crisis amplification for drop in consumption:*

$$\left.\frac{dq_0}{d\kappa}\right|_{\kappa=0} = \begin{cases} \geq 0 \textit{ if } \alpha \leq (1-\beta)\gamma \\ < 0 \textit{ if } \alpha > (1-\beta)\gamma \end{cases} , \quad \left.\frac{dc_0/c_{-1}}{d\kappa}\right|_{\kappa=0} < 0.$$

The first part of this result is obtained by substituting $\kappa = 0$ in Eq. 13-16 and 20. The second part is obtained after differentiating Eq. 20 with respect to κ and evaluating it at $\kappa = 0$. QED.

Proposition 3. *When the LtV limit is at the maximum and the equilibrium is unique ($\kappa = 1$ and $\alpha = (1-\beta)\gamma$) then:*

- *One-to-one mapping from shock to drop in consumption, $\frac{c_0}{c_{-1}} = \gamma$, and there is a slight price inflation, $\frac{q_0}{q_{-1}} = 1 + \left(\frac{1-\beta}{\beta}\right)(1-\gamma) > 1$.*
- *Crisis amplification from decreases in the LtV from one in both the asset price and consumption:*

$$\left.\frac{dq_0}{d\kappa}\right|_{\kappa=1} > 0, \quad \left.\frac{dc_0/c_{-1}}{d\kappa}\right|_{\kappa=1} > 0.$$

The proof of this result is analogous to how we obtained Proposition 2 evaluating the derivatives at $\kappa = 1$ and $\alpha = (1 - \beta)\gamma$. QED.

Proposition 4. *When $\alpha = (1 - \beta)\gamma$ and hence the equilibrium is unique over all $\kappa \in [0, 1]$, then price deflation happens when $\kappa < \frac{1-\gamma(1-\beta)}{1-\gamma(1-\beta)^2}$ and inflation when κ is above such threshold.*

This result is obtained by substituting $\alpha = (1 - \beta)\gamma$ and comparing q_{-1} with q_0 from Eq. 20, then since q_0 is increasing in κ (Proposition 2) we solve for the κ such that $\frac{q_0}{q_{-1}} = 1$ to obtain the threshold. QED.

The derived propositions further elucidate the implications of the model. For instance, the finding that the debt-to-endowment ratio is constrained by the shock size under a unique equilibrium underscores the importance of managing leverage in the economy. Moreover, the analysis shows that the effects of increasing the LtV limit are nuanced: while it can dampen crises in some cases, it may amplify them in others, depending on the capital share and current LtV level. This non-monotonic behavior is largely driven by the interaction between asset prices and collateral constraints (see Eq. 6). Specifically, when the LtV limit increases, two opposing forces come into play. On one hand, higher LtV limits allow for greater indebtedness, making the economy more vulnerable to shocks and amplifying the drop in consumption. On the other hand, a looser collateral constraint mitigates the downward pressure on asset prices, dampening the overall impact of the shock. This result emphasizes the trade-offs inherent in financial regulation and the need for careful calibration of LtV limits to balance economic growth and financial stability.

Overall, this section demonstrates the power of analytical modeling in uncover-ing the complex interplay between collateral constraints, asset prices, and macroeconomic outcomes. The insights gained from the closed-form solutions provide a solid foundation for understanding the broader dynamics explored in the more complex, stochastic version of the model.

2.2 Numerical Exercise

To further dissect the mechanics of our model, we analyze numerically a version of the model without uncertainty, allowing us to isolate the core dynamics at play. This approach helps in understanding the fundamental behavior of the model under different levels of the loan-to-value (LtV) limit, κ, particularly focusing on the existence and nature of equilibria.

The parameters used for the numerical exercise are shown in Table 1. These are standard values in the literature.

Table 1: Parameter Values

Parameter		Value
β, $(R = \beta^{-1})$	discount factor	0.94
y	total endowment	1.0
γ	drop endowment	0.95
α	capital share	$\in \{(1-\beta)\lambda = 0.057,\ 0.15,\ 0.20\}$
κ	LtV limit	$\in [0,\ 1]$

The results presented in the following figures highlight three distinct scenarios: cases where the model yields a unique equilibrium, multiple equilibria, or no equilibrium at all. These scenarios are critical for understanding the potential stability or instability of the economy under varying financial constraints.

Unique Equilibrium Case

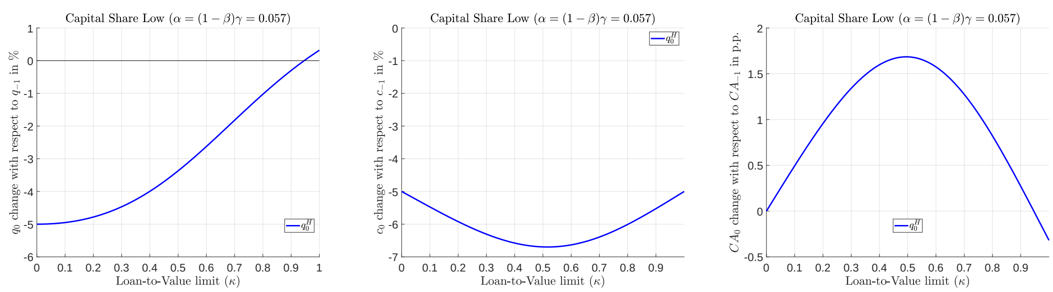
Figure 1 specifically illustrates the case where the capital share α, is equal to $(1-\beta)\gamma$, the maximum value that guarantees a unique equilibrium across all LtV limits ranging from 0 to 1. This particular setting allows us to observe how the economy responds when the LtV limit is varied in a controlled, predictable environment, free from the complexities introduced by uncertainty.

Panel a) of Figure 1 demonstrates a monotonic dampening in the decline of asset prices as the LtV limit increases. This behavior aligns with the intuition that higher LtV limits, by loosening borrowing constraints, mitigate the downward pressure on asset prices. As the collateral constraint becomes less binding, the market is able to absorb shocks more efficiently, preventing a sharp drop in asset values.

Panel b) shows a U-shaped response in the consumption drop as the LtV limit increases. This non-monotonic pattern is particularly noteworthy as it reflects the delicate balance between increased borrowing capacity and heightened vulnerability. At lower LtV limits, the economy is less leveraged, and the impact of shocks on consumption is relatively contained. However, as the LtV limit increases, the increased leverage makes the economy more susceptible to shocks, leading to a more pronounced decline in consumption. Beyond a certain point, though, the loosening of constraints allows for greater consumption smoothing, hence the U-shaped pattern.

Panel c) presents the corresponding change in the current account to GDP ratio, which exhibits a mirror image of the U-shaped consumption response. This inverse relationship highlights the trade-offs faced by the economy as it adjusts to shocks under different LtV regimes. When the LtV limit is low, the limited borrowing capacity leads to smaller swings in the current account. As the LtV increases, the economy finances consumption by borrowing more and when the economy becomes overly leveraged, the need to correct this imbalance in the face of adverse shocks results in a sharp reversal, hence the mirrored U-shape.

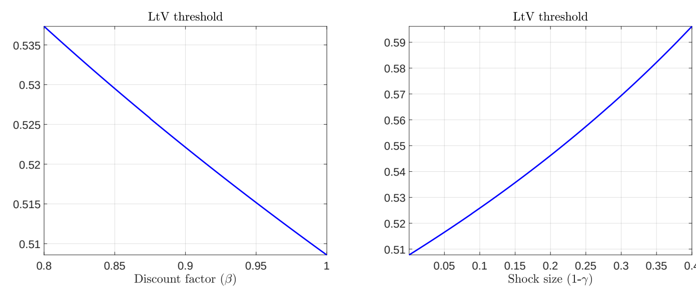
These findings from the unique equilibrium case highlight the critical role that collateral constraints play in shaping the economy's response to shocks. The existence of a unique equilibrium in the setting here indicates a predictable and stable economic environment where policy interventions can be more effectively tailored. The U-shaped consumption response and the mirrored current account behavior further emphasize the nonlinear effects of varying LtV limits, suggesting that while increasing the LtV limit can offer short-term benefits in terms of consumption smoothing, it also introduces longer-term risks associated with higher leverage.



(a) Asset Price (b) Consumption (c) Current Account to GDP ratio

Figure 1: Percent changes during crisis for low capital share (α)

Figure 2 illustrates how the threshold on the LtV limit, which determines the minimum of the U-shaped consumption response, changes for different values of the discount factor and the size of the shock. For LtVs below (above) the threshold, increases in the LtV amplify (dampen) the decline in consumption. Panel a) shows that in more impatient economies the LtV threshold is higher. This means that the interval in which increasing the LtV amplifies the decline in consumption is larger. Similarly, panel b) shows that the same happens in economies with larger shocks.



(a) Discount Factor (b) Shock Size

Figure 2: Comparative Statics

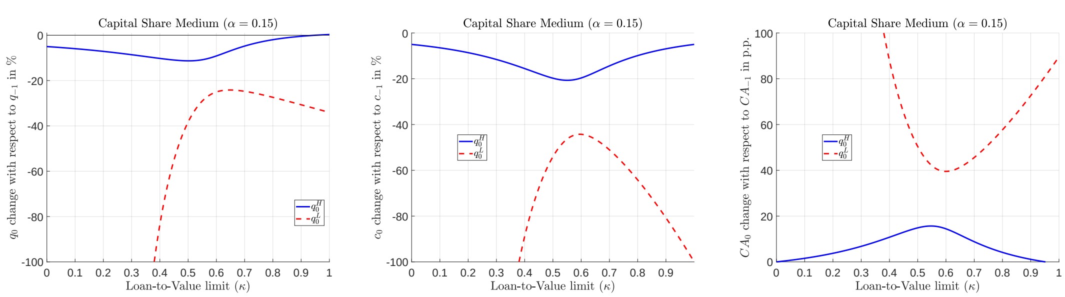
We now explore the behavior of the model when the capital share (α) is set to medium and high levels, specifically 0.15 and 0.20, as illustrated in Figures 3 and 4, respectively. These scenarios allow us to examine how increasing the capital share influences the stability of the economy, particularly in terms of the existence and multiplicity of equilibria.

Multiple Equilibria

In Figure 3, where α is set to 0.15, the model begins to exhibit more complex dynamics compared to the unique equilibrium case. At this medium capital share level, certain values of the LtV limit lead to multiple equilibria. The solid blue lines correspond to the equilibrium obtained with the "high" asset price while the dashed red lines correspond to the equilibrium with the "low" asset price from Eq. 20. Note that the dynamics of the "high" asset price are similar to the dynamics under a unique equilibrium and are more stable relative to the "low" asset price, which generates total asset price collapses for some LtV limits.

This outcome reflects the increased role of capital in the production process, which amplifies the feedback effects between asset prices and collateral constraints. As the capital share rises, the value of collateral becomes more sensitive to fluctuations in asset prices, making the economy more prone to nonlinear and potentially unstable responses to shocks.

In this scenario, the economy can settle into different equilibrium states depending on initial conditions or exogenous shocks. For instance, one equilibrium might be characterized by relatively stable asset prices and consumption, while another might involve more pronounced declines in these variables. This multiplicity of equilibria introduces an element of unpredictability, as small perturbations can push the economy from one equilibrium to another, potentially triggering a financial crisis or exacerbating its effects.



(a) Asset Price (b) Consumption (c) Current Account to GDP ratio

Figure 3: Percent changes during crisis for medium capital share (α)

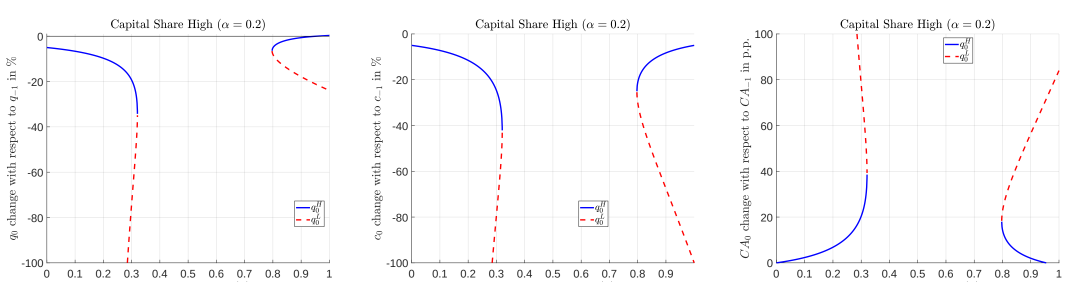
Non-Existent Equilibrium

At certain LtV levels, the model fails to converge to an equilibrium, indicating a breakdown in the standard market-clearing mechanism. This situation could arise when the tightening of collateral constraints due to falling asset prices creates a feedback loop so severe that the economy cannot stabilize, reflecting an extreme form of financial fragility.

Figure 4 further explores this dynamic by increasing the capital share to 0.20. At this elevated level, the economy becomes even more sensitive to changes in the LtV limit, with a broader range of LtV values leading to multiple equilibria or non-existent equilibria.

The high capital share scenario underscores the dangers of high leverage in an economy where capital plays a dominant role. As α increases, the productive capacity of the economy becomes more reliant on capital, and thus, more exposed to fluctuations in asset prices. The tightening of collateral constraints in response to declining asset values can quickly spiral out of control, leading to severe disruptions in consumption and external balances.

In these high α scenarios, the model demonstrates how elevated capital intensity can exacerbate financial instability, particularly in the presence of loose borrowing constraints (high LtV limits where $\kappa > (1-\beta)\frac{\gamma}{\alpha}$). The combination of high leverage and a significant capital share creates conditions where financial markets can no longer clear, resulting in a collapse of asset prices, a sharp contraction in consumption, and potentially, a failure of the economy to reach an equilibrium.



(a) Asset Price (b) Consumption (c) Current Account to GDP ratio

Figure 4: Percent changes during crisis for high capital share (α)

The analysis of the model under medium and high capital share levels provides critical insights into the conditions under which financial markets may become unstable. The presence of multiple equilibria and the potential for non-existent equilibria highlight the risks associated with high leverage and significant capital intensity in the economy.

From a policy perspective, these findings suggest that regulators need to be particularly cautious when allowing high LtV limits in economies with a high capital share. While higher LtV limits can facilitate investment and growth during stable periods, they can also lead to multiple and potentially unstable equilibria, increasing the risk of financial crises.

In economies where capital plays a major role, policies that aim to moderate leverage and ensure more robust collateral frameworks could help prevent the onset of these unstable equilibria. Additionally, understanding the conditions that lead to non-existent equilibria can inform the design of safety nets and intervention mechanisms that could prevent the economy from spiraling into a state where markets fail to clear altogether.

Overall, the findings from these scenarios underscore the importance of considering the interaction between capital intensity and financial constraints in macroeconomic models. By highlighting the potential for multiple or non-existent equilibria, this analysis adds an important dimension to our understanding of financial stability and the factors that can precipitate or mitigate financial crises. This analysis also reinforces the importance of considering nonlinear dynamics in policy design. Policymakers need to be aware that increasing financial flexibility through higher LtV limits may only be beneficial up to a point. Beyond that, the increased vulnerability can lead to more severe downturns in the event of adverse shocks. The insights gained from this simplified model without uncertainty provide a foundational understanding that complements the more complex dynamics observed in the stochastic version of the model, which we present in the next section.

3 Model with Uncertainty

There is not much we can do analytically when there is uncertainty in the economy. However, in this section we compute an extension of the model where the income endowment, y, is stochastic and follows an AR(1) Markov process. Specifically, $\log y_t = (1-\rho)\log(y_{ss}) + \rho\log(y_{t-1}) + \sigma_y \epsilon_t$, where $\epsilon_t \sim N(0,1)$. The additional parameters will take the values $\rho = 0.70$ and $\sigma_y = 0.025$, which are common values used in the literature. The rest of the parameters stay the same as in Table 1 with the exception of the interest rate which is lowered such that $R\beta < 1$ to guarantee a non-degenerate limiting wealth distribution: $R = 1.03$.[3] The recursive representation of the problem of the household becomes:

$$\begin{aligned} V(a,b,s) &= \max_{\{c,b',a'\geq 0\}} u(c) + \beta E_{s'|s}[V(a',b',s')] \quad s.t. \qquad (21) \\ c + R^{-1}b' + qa' &= (1-\alpha)y(s) + a(\alpha y(s) + q) + b \\ R^{-1}b' &\geq -\kappa q a'. \end{aligned}$$

[3]To efficiently solve the model with uncertainty, we rely on the FiPIt global algorithm proposed by Mendoza and Villalvazo (2020), which converges for parameters close to the parameter regions that deliver a unique equilibrium derived in Section 2.1.1. Hence, to analyze the role of a LtV limit between 0 and 1 we used a capital income share below the uniqueness threshold, specifically $\alpha = 0.02$.

3.1 Quantitative Results

In this section, we analyze the quantitative implications of varying the loan-to-value (LtV) limit, κ, in an economy where the households are exposed to an aggregate shock to their endowment. The representative agent framework allows us to model the aggregate implications of endowment shocks, providing a clear lens through which to observe the dynamics of financial crises under different levels of collateral constraints.

To explore these dynamics, we solve, simulate, and derive impulse response functions (IRFs) starting from the stochastic steady state for a range of LtV limits. Figure 5 illustrates the impact of a 2 standard deviation shock on key macroeconomic variables—namely asset prices, consumption, and the current account to GDP ratio—across different values of κ. The solid red lines in the figure depict the changes in these variables on impact.

Our findings reveal a non-monotonic relationship between the LtV limit and the economy's response to shocks, consistent with the theoretical predictions outlined in the earlier sections using the deterministic model. Specifically, as κ increases, we observe that the effects of the shock on asset prices, consumption, and the current account vary in a nonlinear fashion. This non-monotonicity underscores the complex role that financial constraints play in amplifying or dampening the impact of shocks in the presence of aggregate uncertainty.

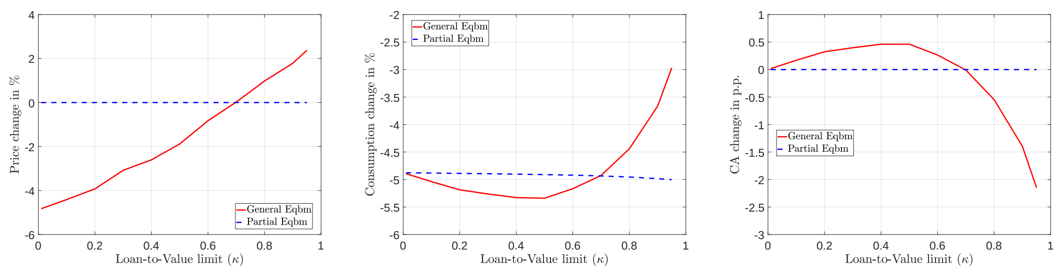
A crucial aspect of these results is the role of the economy's stochastic steady state, where $\beta R < 1$. This condition ensures that the economy operates in a constrained state, making the simulations directly comparable to the analytical results obtained earlier. The constrained state reflects the inherent vulnerability of the economy to shocks, particularly when borrowing limits are binding, and the LtV limit is a pivotal factor in determining the severity of the economy's response.

To further understand the mechanisms driving these results, we contrast the general equilibrium effects with partial equilibrium responses. The blue dashed lines in Figure 5 represent the outcomes when the asset price is held fixed at its stochastic steady state level, effectively isolating the role of pecuniary externalities. In this partial equilibrium scenario, the decline in consumption is relatively flat across different LtV limits, indicating that without the feedback loop generated by fluctuating asset prices, the economy's response to shocks is more muted.

This comparison highlights the critical importance of pecuniary externalities in shaping the overall dynamics. The non-monotonic consumption behavior observed in the general equilibrium setting is largely driven by the interaction between asset prices and collateral constraints. Specifically, when the LtV limit increases, two opposing forces come into play. On one hand, higher LtV limits allow for greater indebtedness, making the economy more vulnerable to shocks and amplifying the drop in consumption. On the other hand, a looser collateral constraint mitigates the downward pressure on asset prices, dampening the overall impact of the shock.

In economies with low LtV limits, the vulnerability effect dominates, leading to a more pronounced decline in consumption. Conversely, in economies with high LtV limits, the mitigating effect on asset prices prevails, resulting in a less severe consumption decline. This interplay between vulnerability and mitigation is crucial for understanding the nonlinear responses of economies to financial shocks and underscores the nuanced role of LtV limits in crisis dynamics.

Overall, these quantitative results not only reinforce the theoretical predictions but also provide a richer understanding of the mechanisms at play. By illustrating how the LtV limit influences the economy's response to shocks through both direct and indirect channels, this analysis contributes to a more comprehensive view of the factors that drive financial instability and offers insights into the potential policy implications of varying collateral constraints.



(a) Asset Price (b) Consumption (c) Current Account to GDP ratio

Figure 5: Change on impact after a 2 s.d. shock

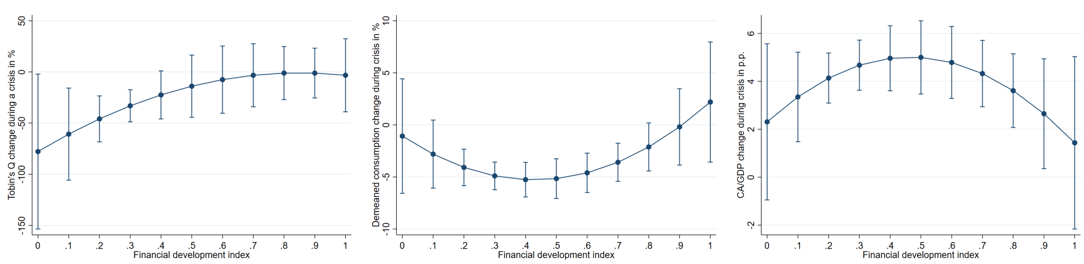
4 Model Validation

To empirically validate the theoretical predictions discussed in this paper, we construct a comprehensive panel database encompassing Sudden Stop episodes from Bianchi and Mendoza (2020), alongside a financial development index developed by Svirydzenka (2016) and macroeconomic aggregates sourced from the World Bank's World Development Indicators (WDI), The World Bank (2024). This dataset enables us to examine the relationship between financial development and key macroeconomic outcomes during financial crises, thereby providing a real-world test of our model's implications.

The validation exercise focuses on a descriptive regression analysis, where we investigate the impact of varying levels of financial development on asset prices, consumption growth, and the change in the current account to GDP ratio. Specifically, we estimate the following quadratic regression model:

$$Dependent_{i,t}\% = \beta_0 + \beta_1 FD_{i,t} + \beta_2 FD_{i,t}^2 + \epsilon_{i,t}. \tag{22}$$

Here, the dependent variables are the percentage changes in asset prices, consumption growth, and the current account to GDP ratio, while $FD_{i,t}$ represents the financial development index. This quadratic specification allows us to capture potential nonlinear effects, consistent with the theory's prediction that the impact of financial development is not linear but rather exhibits a complex, non-monotonic relationship.



(a) Asset Price (b) Consumption (c) Current Account to GDP ratio

Figure 6: Regression estimates with 90 percent confidence bands

As illustrated in Figure 6, the empirical findings align closely with the theoretical predictions. The regression results show a monotonic increase in asset price changes with higher levels of financial development, which supports the model's assertion that more developed financial systems are better equipped to absorb shocks without triggering severe asset price declines. Moreover, the data reveal a U-shaped relationship between consumption growth and financial development, echoing our theoretical findings that economies with intermediate levels of financial development are more vulnerable to consumption declines during crises. In contrast, economies at either low or high ends of the financial development spectrum exhibit less severe consumption declines, due to differing mechanisms of financial constraint tightening.

Similarly, the change in the current account to GDP ratio displays a mirror image of the U-shaped relationship observed for consumption growth. This finding underscores the model's implication that financial development influences the external balance adjustment process during crises, with intermediate levels of development leading to more pronounced swings in the current account.

These results provide robust empirical support for the theoretical framework outlined in this paper. They highlight the importance of considering nonlinearities in the relationship between financial development and macroeconomic outcomes during crises, and reinforce the idea that the effectiveness of financial development in mitigating crisis impacts is highly context-dependent. By validating our model's predictions with real-world data, this section bridges the gap between theory and practice, offering insights that are practically relevant for policymakers.

5 Conclusion

This paper has delved into the intricate dynamics of financial crises, particularly focusing on the role of loan-to-value borrowing constraints in shaping dynamics of equilibrium asset pricing models with occasionally binding collateral constraints.

By elucidating the Fisherian debt-deflation mechanism inherent in these models, we have examined the implications of a wide range of values for loan-to-value limits and uncovered a critical threshold that determines asset price movements, consumption patterns, and current account behaviors. We found that increasing the LtV limit leads to nonlinear effects on key macroeconomic variables such as asset prices, consumption, and the current account. Specifically, a unique equilibrium exists when the capital share is set at the level that guarantees stability across all LtV limits. In this scenario, higher LtV limits mitigate declines in asset prices and create a U-shaped consumption response, balancing the benefits of increased borrowing capacity with the risks of heightened leverage.

However, when capital share increases, the model exhibits more complex dynamics, including the possibility of multiple or non-existent equilibria. This reflects the greater sensitivity of asset prices to shocks, especially in highly leveraged economies. These findings underscore the risks associated with high leverage and loose borrowing constraints, particularly in economies with significant capital intensity. The presence of multiple equilibria or the failure of markets to clear altogether suggests that policy measures should carefully moderate LtV limits, balancing the benefits of credit expansion with the need for financial stability.

These theoretical findings are validated by cross-country relationship between level of financial development and severity of consumption decline during financial crises. In particular, a U-shaped relationship between consumption growth and financial development index emerges during crises. These results provide robust empirical support for the theoretical framework outlined in this paper. They highlight the importance of considering nonlinearities in the relationship between financial development and macroeconomic outcomes during crises, and reinforce the idea that the effectiveness of financial development in mitigating crisis impacts is highly context-dependent. By validating our model's predictions with real-world data, the empirical findings bridges the gap between theory and practice, offering insights that are practically relevant for policymakers.

References

BENIGNO, G., H. CHEN, C. OTROK, A. REBUCCI, AND E. R. YOUNG (2013): "Financial crises and macro-prudential policies," *Journal of International Economics*, 89(2), 453–470.

——— (2016): "Optimal capital controls and real exchange rate policies: A pecuniary externality perspective," *Journal of Monetary Economics*, 84, 147–165.

BIANCHI, J. (2011): "Overborrowing and systemic externalities in the business cycle," *American Economic Review*, 101(7), 3400–3426.

BIANCHI, J., AND E. G. MENDOZA (2018): "Optimal time-consistent macroprudential policy," *Journal of Political Economy*, 126(2), 588–634.

——— (2020): "A fisherian approach to financial crises: Lessons from the sudden stops literature," *Review of Economic Dynamics*, 37, S254–S283.

BORDO, M. D., AND C. M. MEISSNER (2015): "Growing up to stability? Financial globalization, financial development and financial crises," Discussion paper, National Bureau of Economic Research.

DEVEREUX, M. B., E. R. YOUNG, AND C. YU (2019): "Capital controls and monetary policy in sudden-stop economies," *Journal of Monetary Economics*, 103, 52–74.

JEANNE, O., AND A. KORINEK (2019): "Managing credit booms and busts: A Pigouvian taxation approach," *Journal of Monetary Economics*, 107, 2–17.

Jensen, H., S. H. Ravn, and E. Santoro (2018): "Changing credit limits, changing business cycles," *European Economic Review*, 102, 211–239.

Levine, R. (1997): "Financial development and economic growth: views and agenda," *Journal of Economic Literature*, 35(2), 688–726.

Loayza, N. V., and R. Ranciere (2006): "Financial development, financial fragility, and growth," *Journal of Money, Credit and Banking*, pp. 1051–1076.

Lorenzoni, G. (2008): "Inefficient credit booms," *The Review of Economic Studies*, 75(3), 809–833.

Mendoza, E. G. (2010): "Sudden stops, financial crises, and leverage," *American Economic Review*, 100(5), 1941–1966.

Mendoza, E. G., and S. Villalvazo (2020): "FiPIt: A simple, fast global method for solving models with two endogenous states & occasionally binding constraints," *Review of Economic Dynamics*, 37, 81–102.

Ottonello, P., D. J. Perez, and P. Varraso (2022): "Are collateral-constraint models ready for macroprudential policy design?," *Journal of International Economics*, 139, 103650.

Schmitt-Grohé, S., and M. Uribe (2017): "Adjustment to small, large, and sunspot shocks in open economies with stock collateral constraints," *Ensayos sobre Política Económica*, 35(spe82), 2–9.

——— (2021): "Multiple equilibria in open economies with collateral constraints," *The Review of Economic Studies*, 88(2), 969–1001.

SVIRYDZENKA, K. (2016): *Introducing a new broad-based index of financial development*. International Monetary Fund.

THE WORLD BANK (2024): "World development indicators," .